Putnam training problems

2017 - Set 3

Problem 1. Find all natural numbers n such that $n!$ ends in exactly 1000 zeroes.

Problem 2. Prove that if n is a natural number then 2^n does not divide $n!$.

Problem 3. Prove that if $n > 1$ is a natural number then $\frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$ is not an integer.

Problem 4. Find all solutions in the positive integers to the equation $x^{x+y} = y^{y-x}$.

Problem 5. Determine if there exists a function $f : \mathbb{N} \to \mathbb{N}$ such that is strictly increasing, and for which $f(1) = 2$ and $f(f(n)) = f(n) + n$ for all $n \in \mathbb{N}$.

Problem 6. Find all positive integers n such that $\frac{n^3-3n+4}{2n-1}$ is an integer.

Problem 7. Prove that there is a Fibonacci number whose last ten digits are zero

Problem 8. Prove that for every positive integer k there is a Fibonacci number with at least k distinct prime divisors.

Problem 9. Show that the equation

$$x^{2008} + 2008! = 21^y$$

does not have solutions in the integers.

Problem 10. For any prime p prove that there are infinitely many multiples of p whose last ten digits are different.

Problem 11. Prove that the equation

$$2^x + 3 = z^3$$

has no solutions in the integers.

Problem 12. Let n be a positive integer. Find the greatest common denominator of $n! + 1$ and $(n+1)!$.

Problem 13. Prove that if x is an integer, then $x^2 + 1$ has no divisors of the form $4k + 3$.

Problem 14. Prove that there is an infinite number of prime of the form $4k + 1$.